Optical experimental solution for the multi-way number partitioning problem and its application to computing-power scheduling

Jingwei Wen[1],[*] Zhenming Wang[2], Zhiguo Huang[1], Dunbo Cai[1], Bingjie Jia[2], Chongyu Cao[2], Yin Ma[2], Hai Wei[2], Kai Wen[2],[†] and Ling Qian[1‡]
[1] *China Mobile (Suzhou) Software Technology Company Limited, Suzhou 215163, China and*
[2] *Beijing QBoson Quantum Technology Co., Ltd., Beijing 100015, China*

Quantum computing is an emerging technology that is expected to achieve an exponential improvement in computing power. Its theoretical basis and application scenarios have been extensively studied and explored in recent years. In this work, we propose efficient quantum algorithms suitable for computing-power scheduling problems in the cloud rendering domain, which can be regarded mathematically as a generalized form of the typical NP-complete problem – multi-way number partitioning problem. In our algorithm, the matching pattern between tasks and computing resources with minimal completion time or optimal load balancing is encoded into the ground-state of the Hamiltonian, and then solved by the optical coherent Ising machine, a practical quantum computing device with at least 100 qubits. The experimental results show that the proposed quantum scheme can achieve significant acceleration and save 97% solving time on average compared with classical algorithms, which prove the computational advantages of optical quantum devices in solving combinatorial optimization problems. Our algorithmic and experimental work advances the utilization of quantum computers to solve specific NP problems and expands practical application scenarios.

I. INTRODUCTION

The development of quantum technology is of great scientific significance and social value, which is expected to have a major impact on traditional technology and trigger technological revolution and industrial transformation [1–4]. Quantum computing, as the frontier of quantum technology, is dedicated to using the principles of quantum mechanics to calculate and simulate complex systems [5–7]. Due to its potential advantages in processing large amounts of data quickly and efficiently, quantum computers are expected to play an important role in secure encryption [8, 9], database search [10, 11], machine learning [12, 13] and many other scenarios that are intractable with classical computers [14–20].

By harnessing the power of quantum computers, people can also quickly optimize the scheduling processes of personnel and equipment in order to maximize efficiency and minimize costs, in scenarios such as communication networks [21, 22], healthcare [23], transportation [24–26], and complex supply chain management [27–32]. In addition, quantum computing has the potential to revolutionize the field of computing-power scheduling in cloud computing [33–35], which needs to search the large solution-space for the optimal configurations for allocating computing resources to different tasks with improved performance and efficiency.

In this work, focusing on a specific application scenario about computing-power scheduling problem in cloud-rendering domain, which can mathematically modeled as a generalized form of the multi-way number partitioning problem, an important NP-complete problem [36–44], we propose two quantum algorithms from different optimization perspectives and present an experimental solution demonstration on the optical coherent Ising machine (CIM) [45–53], an over 100-qubit quantum computing device. The experimental results show that our quantum scheme can realize a significant quantum acceleration, saving 97% solving time on average compared with classical simulated annealing (SA) and tabu search algorithms [54]. As the first experimental demonstration of quantum algorithm for the generalized multi-way number partitioning problem in optical systems, our work shows the acceleration of quantum computers relative to classical techniques for a specific NP-complete problem, and also explores a new application scenario for quantum computing.

This paper is organized as follows: we first introduce some preliminaries in the Sec II, followed by a description to the proposed quantum algorithms in Sec III, which transform the scheduling of computing-power resources into an optimization problem. In Sec IV, an experimental demonstration of the quantum algorithm is carried out on a practical optical quantum computer and comparisons with classical algorithms are offered. Finally, we give a conclusion in sec V.

II. PRELIMINARIES

A. Background and Mathematical Reduction

Image rendering in the field of film and television is an important application scene of cloud computing [55], as shown in figure 1. In general, such a scenario can be expressed as: *the client submits a rendering task with certain requirements and the service providers need to find*

[*]Electronic address: wjw17@tsinghua.org.cn
[†]Electronic address: wenk@boseq.com
[‡]Electronic address: qianling@cmss.chinamobile.com

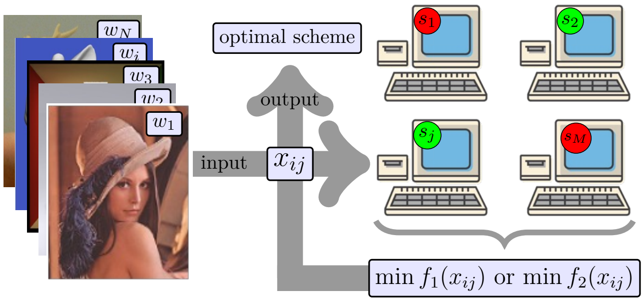

FIG. 1: A schematic of the computing-power scheduling process in cloud-rendering. There are N tasks in the waiting list to be allocated to M machines. The expected duration of task $i \in \{1, 2, \cdots, N\}$ is w_i and the start of available time for machine $j \in \{1, 2, \cdots, M\}$ is s_j. By minimizing the proposed optimization functions $f_1(x_{ij})$ or $f_2(x_{ij})$, the objective schemes (x_{ij}) with minimal completion time or optimal load balancing can be found.

the optimal schemes to invoke computing resources. The basic idea is searching the minimum number of servers under constraints, which is realized by presetting a machine number first, and then apply various heuristic algorithms to determine the current state (completion time, load balancing, and so on) for satisfiability comparison. However, due to the increasing amount of data, the optimal task-server scheduling scheme cannot be effectively obtained in the large-scale dynamic cloud-rendering and redundant rendering is a common phenomenon. This would lead to the mismatch between server and rendering tasks, causing computing resource waste and rendering efficiency reduction.

Mathematically, this process can be modeled as a generalized multi-way number partitioning problem [42–44], which is a typical NP-complete problem [37]. The multi-way number partitioning problem is defined as dividing/-partitioning a given set $S = \{\cdots, a_i, \cdots\}$ of positive integer into k subsets to make the subset-sum as nearly equal as possible. For the widely studied $k = 2$ case, number partitioning problem (NPP) can be formulated as an optimization problem to minimize the difference

$$D(A) = \Big| \sum_{a_i \in A} a_i - \sum_{a_i \in S \setminus A} a_i \Big| \tag{1}$$

between subset A and complementary set $S \setminus A$ [40, 41]. Here we extend the balanced bipartition ($k = 2$) to multi-partitioned cases ($k \geq 2$) without constraining the elements to be integers. In addition, the target sum of each subset can be set differently, which means that the set S is partitioned into several subsets with unequal sums. Through these extensions, we can establish a correspondence between such a generalized multi-way number partitioning problem and computing-power scheduling problem in cloud-rendering domain, which is detailed in Section III.

B. QUBO and Ising Model

Here we briefly introduce the quadratic unbounded binary optimization (QUBO) problem and its relationship with Ising model. It is well-established that many canonical NP-hard and NP-complete problems can be transformed into combinatorial optimization form [56]. A large class of these optimization problems can be expressed either in QUBO form with binary variables of $\{0, 1\}$ basis, or in $\{-1, 1\}$ basis with spin variables of Ising model. These two forms are equivalent and can be easily converted [57]. To be specific, the mathematical form of QUBO problem is expressed as follows

$$f_{\mathrm{QUBO}}(x) = \sum_{i \leq j} q_{ij} x_i x_j = x^T Q x \tag{2}$$

where $x = \{x_i\}$ is the binary variable vector to be solved, and QUBO matrix $Q = \{q_{ij}\}$ represents the quadratic coefficients. The objective solution is

$$x^* = \arg\min_x f_{\mathrm{QUBO}}(x) \tag{3}$$

By transforming variables as $x_i \to (I + \sigma_i)/2$ where the σ_i is spin variable, the optimization functions can be presented with an Ising model. Then the target solution is encoded in the ground-states of the Hamiltonian

$$H_{Ising}(\sigma) = -\sum_{i,j} J_{ij} \sigma_i \sigma_j - \sum_i h_i \sigma_i \tag{4}$$

where J_{ij} and h_i are the quadratic and linear coefficients, respectively. In our experimental system, the optical CIM platform can map the target QUBO problem to an all-to-all connectivity Ising Hamiltonian with programmable parameters, and the optimal solution can be obtained by controllable quantum phase transition processes.

Below, we introduce two quantum optimization algorithms that model the computing-power scheduling (generalized multi-way number partitioning) problem as a QUBO problem, and provide an optical experimental demonstration of the algorithm, together with comparisons to classical algorithms.

III. ALGORITHMS

In this part, we propose two quantum algorithms that formulate the generalized multi-way number partitioning problem as a QUBO problem from different optimization perspectives, and take the scheduling processes in cloud-rendering domain as a specific application scenario to illustrate.

We first introduce a series of binary variables x_{ij} to represent the state of matching between N tasks and M machines. Specifically, we set $x_{ij} = 1$ if task $i \in \{1, 2, \cdots, N\}$ is performed/started on machine $j \in$

$\{1, 2, \cdots, M\}$, and $x_{ij} = 0$ otherwise. The expected duration of task i serves as a weight number w_i and the total time of tasks is $\mathcal{W} = \sum_{i=1}^{N} w_i$. The completion time for performing tasks on machine j is $c_j = \sum_{i=1}^{N} w_i x_{ij} + s_j$, where s_j is the idle start time for machine j, satisfying $\sum_{j=1}^{M} s_j = \mathcal{S}$. To ensure task i is only assigned to one machine, we can introduce constraint

$$\sum_{j=1}^{M} x_{ij} = 1 \tag{5}$$

Then we have $\sum_{j=1}^{M} c_j = \mathcal{W} + \mathcal{S}$. First, we propose an optimization function for minimizing the completion time of the whole task, which is equivalent to determine the minimum of the maximum values of c_j

$$\min_{x_{ij}} \max_{j} \{c_1, c_2, \cdots, c_j, \cdots, c_M\} \tag{6}$$

By introducing variable u with $u \geq c_j$ $\forall j$, the original problem can be changed to finding the minimum values of an optimization function $\min f_1(x_{ij}) = \min u$. Then the inequality can be transformed into equality by slack-variables $v_j^{(s)} = u - c_j$ $\forall j$, and they can be expressed with binary expansion as

$$u = \sum_{l=0}^{L-1} 2^l u_l^{(s)} \geq 0, \;\; v_j^{(s)} = \sum_{l=0}^{L-1} 2^l v_{jl}^{(s)} \geq 0 \tag{7}$$

where parameter L is related to precision, while $u_l^{(s)}$ and $v_{jl}^{(s)}$ take values of zero or one. Inserting all the constraints into the objective function, we can get the optimization function for the min-max problem

$$\begin{aligned} f_1(x_{ij}) = & \sum_{l=0}^{L-1} 2^l u_l^{(s)} + \beta_1 \sum_{i=1}^{N} (\sum_{j=1}^{M} x_{ij} - 1)^2 \\ & + \beta_2 \sum_{j=1}^{M} (\sum_{l=0}^{L-1} 2^l (u_l^{(s)} - v_{jl}^{(s)}) - \sum_{i=1}^{N} w_i x_{ij} - s_j)^2 \end{aligned} \tag{8}$$

where β_1 and β_2 are penalty coefficients. The number of variables used here is $(NM + L + ML)$. Due to the introduction of slack-variables, improvement of accuracy depends on the increase of encoding bits, which is not conducive to the application of algorithm on the noisy intermediate-scale quantum (NISQ) devices.

Alternatively, we can turn to find the optimal scheme with a balanced load for each machine, which can be obtained by searching a relatively balanced distribution of completion time [58], and it can be transformed into finding the minimum values of a variance-like optimization function

$$f_2(x_{ij}) = \frac{1}{M} \sum_{j=1}^{M} (c_j - \frac{\mathcal{W} + \mathcal{S}}{M})^2 \tag{9}$$

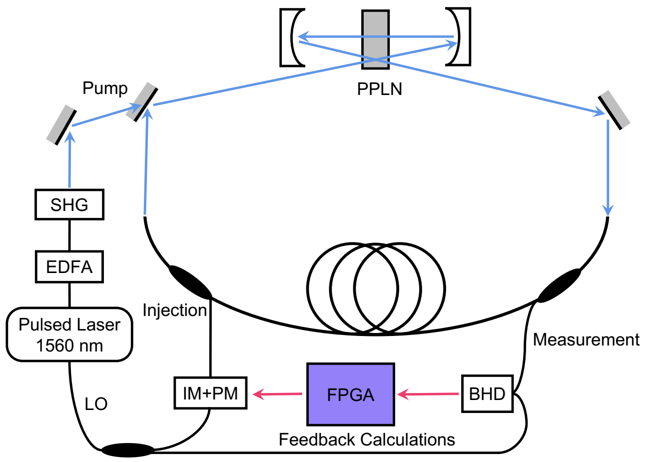

FIG. 2: A schematic diagram of the measurement feedback CIM. EDFA is the erbium-doped fiber amplifier, SHG is the second harmonic generation, BHD is the balanced homodyne detection, IM/PM is the intensity/phase modulator. The pulses of local oscillator (LO) are directly obtained from the master laser.

Combined with the constraint in equation (5), the loss function to be minimized is

$$f_2(x_{ij}) = \frac{1}{M} \sum_{j=1}^{M} (\sum_{i=1}^{N} w_i x_{ij} + s_j - \bar{c})^2 + \beta \sum_{i=1}^{N} (\sum_{j=1}^{M} x_{ij} - 1)^2 \tag{10}$$

where penalty coefficient is β and expected mean value is $\bar{c} = (\mathcal{W} + \mathcal{S})/M$. Compared with optimization function $f_1(x_{ij})$, this variance-like optimization function $f_2(x_{ij})$ only needs MN variables, a big reduction in bits/qubits [59], and it is more consistent with the definition of multi-way number partitioning problem. So we use the latter objective function for experimental demonstration and discussion below. Once we find the optimal $\{0, 1\}$ series for x_{ij}, we can determine what tasks are performed in each machine. Both objective functions constructed here have the QUBO form, and the target solution encoded in the ground-states can be obtained via quantum algorithms like quantum annealing [60, 61] and quantum approximate optimization algorithm [62] on quantum computers. Alternatively, we present the optical experimental solution based on CIM, which can serve as a good physical platform for QUBO problem.

IV. EXPERIMENT

In this part, we present an optical realization and solution of above algorithm based on the CIM. Unlike classical computers that run on semiconductor integrated chips, the CIM system is a hybrid quantum computing platform, using laser pulses in optical fibers as qubits for computation, as shown in figure 2. The optical quantum computer used here is the measurement feedback CIM

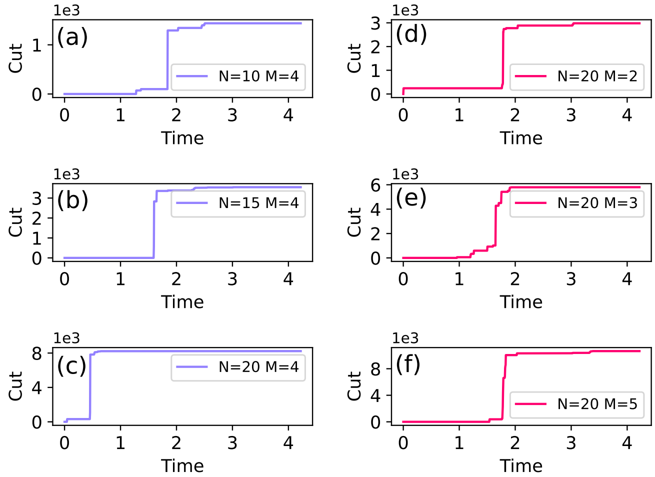

FIG. 3: Cut values with the running time (in millisecond). The panels in the left column (a-c) fix the number of machines as $M = 4$, while panels in the right column (d-f) fix the number of tasks as $N = 20$.

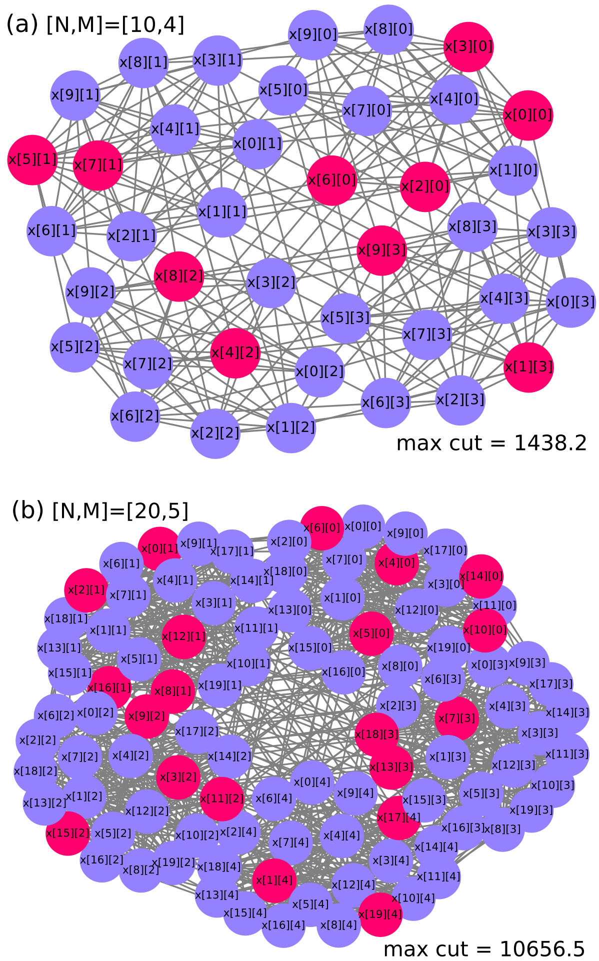

FIG. 4: The graphs and results for experimental demonstration cases with minimum- and maximum-scale. (a) $[N, M] = [10, 4]$ and (b) $[N, M] = [20, 5]$. Notations $x[i][j] = x_{ij}$ here. The node colors indicate different spin results, where red represent $+1$ and blue is -1. The maximum cut value is also marked.

[45] including optical and electrical parts, which is developed by Beijing QBoson Quantum Technology Co., Ltd. [63].

The optical part consists of lasers, amplifiers, PPLN crystals, and fiber ring. The laser is a femtosecond pulsed fiber laser with a locked repetition frequency at 100 MHz. Because the output power (100 mW) of laser is relatively low, an amplification is realized by erbium-doped fiber amplifier (EDFA). The frequency of amplified laser is doubled using PPLN crystal to produce a 780 nm laser, which is used as the pump source to synchronously pump the phase sensitive amplifier (PSA) to form degenerate optical parametric oscillation (DOPO) [64, 65]. There are 211 oscillating pulses in the fiber loop during the calculation, and the time interval of each two pulses is $\Delta t = 10$ ns. Therefore, the transmission time of optical pulses in the loop is $T_c = 2.11$ μs. In addition to the optical part, the electrical part contains field-programmable gate array (FPGA), AD/DA converter, and phase detector. FPGA and high-speed AD/DA are used to measure and control optical pulses based on the interaction intensity, which can realize the interaction between qubits in the Ising model. FPGA by Xilinx used here can support DSP multipliers and on-chip resource storage. The laser output in the fiber ring and the laser of fundamental frequency (1560 nm) are measured by optical balanced homodyne detectors (BHD), which can read out the in-phase amplitude of output pulse.

To test the hardware capability, we run the experiments with cases up to one hundred qubits, and design two groups of experimental schemes, which fixes the machine (or task) number and varies the other parameter. The CIM used for the experiments has a fixed number of simultaneous oscillating pulses in fiber ring for qubits. If the problem scale of the model is below the available qubits, the non-computing qubits will be used to stabilize the system. We assume that the time required for each task and idle time for each machine are both positive integers taken from set $\mathcal{N}^+$. For comparison, we also adopt classical algorithms including SA and tabu search to solve the experimental models, which are run 100 times on a CPU (Intel Core i7-10750H, 2.60 GHz with 16-gigabyte random-access memory) in each problem setup for obtaining the mean values and standard deviations.

As shown in the figure 3, when the power of pump light is gradually increased to the oscillation threshold, phase transition happens and the values of cut increase with running time. The cut value adopted here is the score of the maximum cut problem transformed from the original optimization problem [50], and it is a measure that is anti-linear to the objective function value, that is, the maximization of cut corresponds to the minimization of the objective function. Then the light will trans-

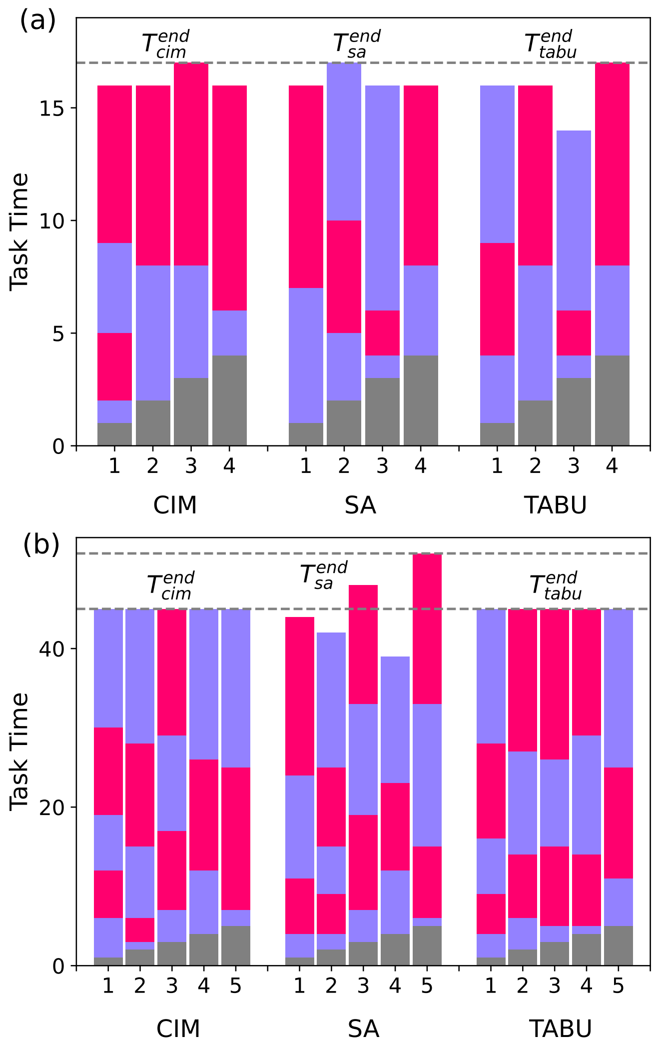

FIG. 5: Scheduling schemes determined by the quantum (CIM) and classical (SA and TABU) algorithms with different problem scales in experiment. (a) $[N, M] = [10, 4]$ and (b) $[N, M] = [20, 5]$. The x-coordinate refers to different machines, while the lengths of red and blue parts indicate the tasks assigned to each machine with corresponding durations, and the gray parts indicate the idle start time for each machine. The makespans (T^{end}_{cim}, T^{end}_{sa} and T^{end}_{tabu}) of the whole task for different solutions are labeled by dotted lines.

form from squeezed vacuum states into coherent states with phase 0 and π respectively, which correspond to the spin states. The loss of such a specific single-mode oscillation is minimal and it corresponds to the ground-state of the Ising Hamiltonian. The figure 4 shows the experimental spin/binary variables results with problem scale $[N, M] = [10, 4]$ and $[20, 5]$, respectively. The red nodes indicate $+1$ ($x_{ij} = 1$) and blue nodes represent -1 ($x_{ij} = 0$) for spins. We can conclude that the nearly all-connected graphs are complicated with high optimization complexity, and experimental output results satisfy the constraints.

We can directly obtain the optimal scheduling method

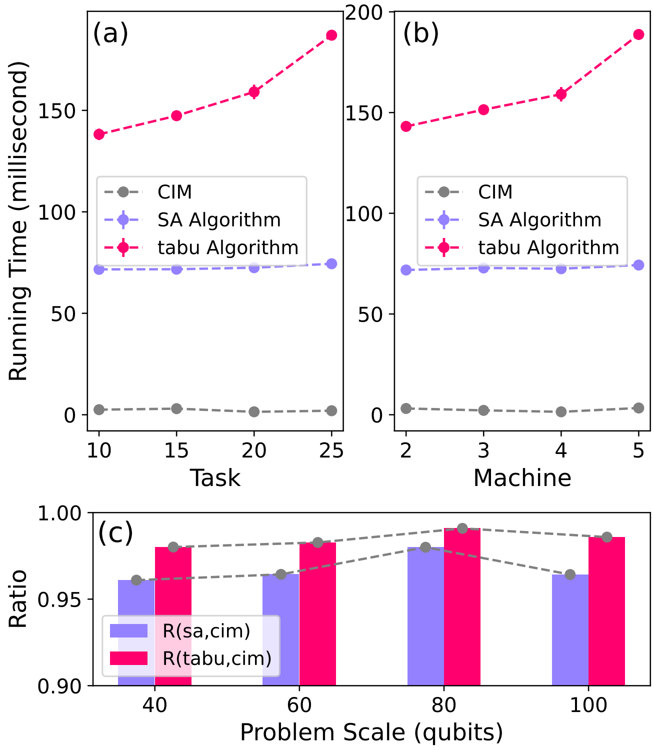

FIG. 6: Running time (in millisecond) of quantum algorithm based on CIM and two classical algorithms (a,b), and the time-saving ratio $R(sa/tabu, cim)$ with problem scale (c). The error bars of classical algorithms are derived from the standard deviations of 100 repetitions. Machine number $M = 4$ is fixed in (a), while in (b) task number is $N = 20$.

between computing-power resources and rendering tasks based on the experimental spin states, as shown in figure 5. The makespans of the optimal allocation scheme are 17 and 45 corresponding to the minimum- and maximum-scale in experimental setup, which can be obtained by quantum and tabu search algorithms. It indicates the feasibility and correctness of quantum algorithm in solving the computing-power scheduling problem. Note that the optimal scheme is not unique due to the parameter setting, namely the ground-state of Ising Hamiltonian is degenerate.

In addition, the running time of the different algorithms is shown in figure 6. Using notations $t_{sa/tabu/cim}$ to represent the running time, we can define a ratio between them as

$$R(sa/tabu, cim) = \frac{t_{sa/tabu} - t_{cim}}{t_{sa/tabu}} \tag{11}$$

to measure the time-saving (or acceleration) capability of quantum algorithm. Given that CIM solution is faster, a larger ratio of $R(sa/tabu, cim) \in (0, 1)$ represents a greater quantum acceleration effect. It can be concluded that the CIM solver is faster than classical algorithms, realizing an average $R(sa, cim) = 96.7\%$ and $R(tabu, cim) = 98.5\%$ time-saving to SA and tabu search algorithms, respectively. On the scale of 100 qubits, quantum solutions can achieve tens of times of acceleration to classical solutions. Furthermore, we can see that while the time of classical SA algorithm does not increase significantly with the scale of problem, it does so at the cost of accuracy, whereas the opposite is true for tabu search algorithm. However, the quantum algorithm can not only guarantee the correctness, but also has a stable running time, which is 2.37 ms on average. The solution time does not improve significantly for CIM with the increase of qubits, so greater advantages can be expected in the larger problem scale.

V. CONCLUSION

Although quantum computers have been proved to be able to surpass classical computers in solving specific problems, developing quantum algorithms for important mathematical problems and actual production scenes is still a field of interest. In this work, we propose two quantum algorithms from different optimization perspectives for solving the computing-power scheduling problems in cloud-rendering domain, which can mathematically be modeled as the generalized multi-way number partitioning ($k \geq 2$) problem, a typical NP-complete problem. Utilizing a 100-qubit optical quantum computing system, we experimentally demonstrate the feasibility and advantage of quantum algorithm, realizing an average 96.7% and 98.5% time-saving to classical SA and tabu search algorithms. The CIM-based quantum computing scheme has a good performance in accuracy and speed, and the running time remains relatively stable with the increase of problem scale, which is expected to gain greater advantages in large-scale problems. It is worth emphasizing that multi-way number partitioning is an important and basic problem, and many other problems like cryptography [40] can also mathematically be reduced to it. Therefore, our work greatly expands the practical application scenarios of quantum computers based on the hardware with tens of thousands of qubits [50] that is available nowadays.

VI. ACKNOWLEDGEMENTS

We acknowledge the support from the National Key R&D Plan (2021YFB2801800).

[1] Yudong Cao, Jonathan Romero, Jonathan P Olson, Matthias Degroote, Peter D Johnson, Mária Kieferová, Ian D Kivlichan, Tim Menke, Borja Peropadre, Nicolas PD Sawaya, et al. Quantum chemistry in the age of quantum computing. *Chemical reviews*, 119(19):10856–10915, 2019.

[2] Dylan Herman, Cody Googin, Xiaoyuan Liu, Alexey Galda, Ilya Safro, Yue Sun, Marco Pistoia, and Yuri Alexeev. A survey of quantum computing for finance. *arXiv preprint arXiv:2201.02773*, 2022.

[3] Felix Tennie and Tim Palmer. Quantum computers for weather and climate prediction: The good, the bad and the noisy. *arXiv preprint arXiv:2210.17460*, 2022.

[4] Sheir Yarkoni, Elena Raponi, Thomas Bäck, and Sebastian Schmitt. Quantum annealing for industry applications: Introduction and review. *Reports on Progress in Physics*, 2022.

[5] Richard P Feynman. Simulating physics with computers. pages 133–153, 2018.

[6] Paul Benioff. The computer as a physical system: A microscopic quantum mechanical hamiltonian model of computers as represented by turing machines. *Journal of statistical physics*, 22(5):563–591, 1980.

[7] Seth Lloyd. Universal quantum simulators. *Science*, 273 (5278):1073–1078, 1996.

[8] Peter W Shor. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. *SIAM review*, 41(2):303–332, 1999.

[9] Peter W Shor and John Preskill. Simple proof of security of the bb84 quantum key distribution protocol. *Physical review letters*, 85(2):441, 2000.

[10] Lov K Grover. Quantum mechanics helps in searching for a needle in a haystack. *Physical review letters*, 79(2): 325, 1997.

[11] Gui-Lu Long. Grover algorithm with zero theoretical failure rate. *Physical Review A*, 64(2):022307, 2001.

[12] Patrick Rebentrost, Masoud Mohseni, and Seth Lloyd. Quantum support vector machine for big data classification. *Physical review letters*, 113(13):130503, 2014.

[13] Hsin-Yuan Huang, Michael Broughton, Masoud Mohseni, Ryan Babbush, Sergio Boixo, Hartmut Neven, and Jarrod R McClean. Power of data in quantum machine learning. *Nature communications*, 12(1):2631, 2021.

[14] Alán Aspuru-Guzik, Anthony D Dutoi, Peter J Love, and Martin Head-Gordon. Simulated quantum computation of molecular energies. *Science*, 309(5741):1704–1707, 2005.

[15] Aram W Harrow, Avinatan Hassidim, and Seth Lloyd. Quantum algorithm for linear systems of equations. *Physical review letters*, 103(15):150502, 2009.

[16] Birger Horstmann, Benni Reznik, Serena Fagnocchi, and J Ignacio Cirac. Hawking radiation from an acoustic black hole on an ion ring. *Physical review letters*, 104 (25):250403, 2010.

[17] Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C Bardin, Rami Barends, Rupak Biswas, Sergio Boixo, Fernando GSL Brandao, David A Buell, et al. Quantum supremacy using a programmable superconducting processor. *Nature*, 574(7779):505–510, 2019.

[18] Han-Sen Zhong, Hui Wang, Yu-Hao Deng, Ming-Cheng Chen, Li-Chao Peng, Yi-Han Luo, Jian Qin, Dian Wu, Xing Ding, Yi Hu, et al. Quantum computational advantage using photons. *Science*, 370(6523):1460–1463, 2020.

[19] Yulin Wu, Wan-Su Bao, Sirui Cao, Fusheng Chen, Ming-Cheng Chen, Xiawei Chen, Tung-Hsun Chung, Hui Deng, Yajie Du, Daojin Fan, et al. Strong quantum computa-

tional advantage using a superconducting quantum processor. *Physical review letters*, 127(18):180501, 2021.

[20] Qingling Zhu, Sirui Cao, Fusheng Chen, Ming-Cheng Chen, Xiawei Chen, Tung-Hsun Chung, Hui Deng, Yajie Du, Daojin Fan, Ming Gong, et al. Quantum computational advantage via 60-qubit 24-cycle random circuit sampling. *Science bulletin*, 67(3):240–245, 2022.

[21] Jaeho Choi, Seunghyeok Oh, and Joongheon Kim. Quantum approximation for wireless scheduling. *Applied Sciences*, 10(20):7116, 2020.

[22] Joongheon Kim, Yunseok Kwak, Soyi Jung, and Jae-Hyun Kim. Quantum scheduling for millimeter-wave observation satellite constellation. In *2021 IEEE VTS 17th Asia Pacific Wireless Communications Symposium (APWCS)*, pages 1–5. IEEE, 2021.

[23] Kazuki Ikeda, Yuma Nakamura, and Travis S Humble. Application of quantum annealing to nurse scheduling problem. *Scientific reports*, 9(1):1–10, 2019.

[24] Alan Crispin and Alex Syrichas. Quantum annealing algorithm for vehicle scheduling. In *2013 IEEE International Conference on Systems, Man, and Cybernetics*, pages 3523–3528. IEEE, 2013.

[25] Tony Tran, Minh Do, Eleanor Rieffel, Jeremy Frank, Zhihui Wang, Bryan O'Gorman, Davide Venturelli, and J Beck. A hybrid quantum-classical approach to solving scheduling problems. In *Proceedings of the International Symposium on Combinatorial Search*, volume 7, pages 98–106, 2016.

[26] Tobias Stollenwerk, Elisabeth Lobe, and Martin Jung. Flight gate assignment with a quantum annealer. In *Quantum Technology and Optimization Problems: First International Workshop, QTOP 2019, Munich, Germany, March 18, 2019, Proceedings 1*, pages 99–110. Springer, 2019.

[27] Tianmin Zheng and Mitsuo Yamashiro. Solving flow shop scheduling problems by quantum differential evolutionary algorithm. *The International Journal of Advanced Manufacturing Technology*, 49:643–662, 2010.

[28] Davide Venturelli, D Marchand, and Galo Rojo. Job shop scheduling solver based on quantum annealing. In *Proc. of ICAPS-16 Workshop on Constraint Satisfaction Techniques for Planning and Scheduling (COPLAS)*, pages 25–34, 2016.

[29] Krzysztof Kurowski, Jan W glarz, Marek Subocz, Rafał Różycki, and Grzegorz Waligóra. Hybrid quantum annealing heuristic method for solving job shop scheduling problem. In *Computational Science–ICCS 2020: 20th International Conference, Amsterdam, The Netherlands, June 3–5, 2020, Proceedings, Part VI 20*, pages 502–515. Springer, 2020.

[30] Sheir Yarkoni, Alex Alekseyenko, Michael Streif, David Von Dollen, Florian Neukart, and Thomas Bäck. Multicar paint shop optimization with quantum annealing. In *2021 IEEE International Conference on Quantum Computing and Engineering (QCE)*, pages 35–41. IEEE, 2021.

[31] AI Pakhomchik, S Yudin, MR Perelshtein, A Alekseyenko, and S Yarkoni. Solving workflow scheduling problems with qubo modeling. *arXiv preprint arXiv:2205.04844*, 2022.

[32] Marc Geitz, Cristian Grozea, Wolfgang Steigerwald, Robin Stöhr, and Armin Wolf. Solving the extended job shop scheduling problem with agvs–classical and quantum approaches. In *Integration of Constraint Programming, Artificial Intelligence, and Operations Research: 19th International Conference, CPAIOR 2022, Los Angeles, CA, USA, June 20-23, 2022, Proceedings*, pages 120–137. Springer, 2022.

[33] Michael Armbrust, Armando Fox, Rean Griffith, Anthony D Joseph, Randy Katz, Andy Konwinski, Gunho Lee, David Patterson, Ariel Rabkin, Ion Stoica, et al. A view of cloud computing. *Communications of the ACM*, 53(4):50–58, 2010.

[34] Sukhpal Singh and Inderveer Chana. A survey on resource scheduling in cloud computing: Issues and challenges. *Journal of grid computing*, 14:217–264, 2016.

[35] Mohit Kumar, Subhash Chander Sharma, Anubhav Goel, and Santar Pal Singh. A comprehensive survey for scheduling techniques in cloud computing. *Journal of Network and Computer Applications*, 143:1–33, 2019.

[36] Sartaj K Sahni. Algorithms for scheduling independent tasks. *Journal of the ACM (JACM)*, 23(1):116–127, 1976.

[37] Ronald V Book. Michael r. garey and david s. johnson, computers and intractability: a guide to the theory of np-completeness. 1980.

[38] H De Raedt, K Michielsen, K De Raedt, and S Miyashita. Number partitioning on a quantum computer. *Physics Letters A*, 290(5-6):227–233, 2001.

[39] Tobias Graß, David Raventós, Bruno Juliá-Díaz, Christian Gogolin, and Maciej Lewenstein. Quantum annealing for the number-partitioning problem using a tunable spin glass of ions. *Nature communications*, 7(1):11524, 2016.

[40] Luca Asproni, Davide Caputo, Blanca Silva, Giovanni Fazzi, and Marco Magagnini. Accuracy and minor embedding in subqubo decomposition with fully connected large problems: a case study about the number partitioning problem. *Quantum Machine Intelligence*, 2(1):4, 2020.

[41] Galit Anikeeva, Ognjen Marković, Victoria Borish, Jacob A Hines, Shankari V Rajagopal, Eric S Cooper, Avikar Periwal, Amir Safavi-Naeini, Emily J Davis, and Monika Schleier-Smith. Number partitioning with grover's algorithm in central spin systems. *PRX Quantum*, 2(2):020319, 2021.

[42] Richard Earl Korf. Multi-way number partitioning. In *Twenty-First International Joint Conference on Artificial Intelligence*, 2009.

[43] Ethan L Schreiber, Richard E Korf, and Michael D Moffitt. Optimal multi-way number partitioning. *Journal of the ACM (JACM)*, 65(4):1–61, 2018.

[44] Yannick Deller, Sebastian Schmitt, Maciej Lewenstein, Steve Lenk, Marika Federer, Fred Jendrzejewski, Philipp Hauke, and Valentin Kasper. Quantum approximate optimization algorithm for qudit systems with long-range interactions. *arXiv preprint arXiv:2204.00340*, 2022.

[45] Yoshitaka Haribara, Yoshihisa Yamamoto, Ken-ichi Kawarabayashi, and Shoko Utsunomiya. A coherent ising machine with quantum measurement and feedback control. *arXiv preprint arXiv:1501.07030*, 2015.

[46] Peter L McMahon, Alireza Marandi, Yoshitaka Haribara, Ryan Hamerly, Carsten Langrock, Shuhei Tamate, Takahiro Inagaki, Hiroki Takesue, Shoko Utsunomiya, Kazuyuki Aihara, et al. A fully programmable 100-spin coherent ising machine with all-to-all connections. *Science*, 354(6312):614–617, 2016.

[47] Takahiro Inagaki, Yoshitaka Haribara, Koji Igarashi, Tomohiro Sonobe, Shuhei Tamate, Toshimori Honjo, Alireza Marandi, Peter L McMahon, Takeshi Umeki,

Koji Enbutsu, et al. A coherent ising machine for 2000-node optimization problems. *Science*, 354(6312):603–606, 2016.

[48] Yoshihisa Yamamoto, Kazuyuki Aihara, Timothee Leleu, Ken-ichi Kawarabayashi, Satoshi Kako, Martin Fejer, Kyo Inoue, and Hiroki Takesue. Coherent ising machines—optical neural networks operating at the quantum limit. *npj Quantum Information*, 3(1):1–15, 2017.

[49] Y Yamamoto, T Leleu, S Ganguli, and H Mabuchi. Coherent ising machines—quantum optics and neural network perspectives. *Applied Physics Letters*, 117(16): 160501, 2020.

[50] Toshimori Honjo, Tomohiro Sonobe, Kensuke Inaba, Takahiro Inagaki, Takuya Ikuta, Yasuhiro Yamada, Takushi Kazama, Koji Enbutsu, Takeshi Umeki, Ryoichi Kasahara, et al. 100,000-spin coherent ising machine. *Science advances*, 7(40):eabh0952, 2021.

[51] Naeimeh Mohseni, Peter L McMahon, and Tim Byrnes. Ising machines as hardware solvers of combinatorial optimization problems. *Nature Reviews Physics*, 4(6):363–379, 2022.

[52] Bo Lu, Lu Liu, Jun-Yang Song, Kai Wen, and Chuan Wang. Recent progress on coherent computation based on quantum squeezing. *AAPPS Bulletin*, 33(1):7, 2023.

[53] Bo Lu, Chen-Rui Fan, Lu Liu, Kai Wen, and Chuan Wang. Speed-up coherent ising machine with a spiking neural network. *Optics Express*, 31(3):3676–3684, 2023.

[54] Habib Youssef, Sadiq M Sait, and Hakim Adiche. Evolutionary algorithms, simulated annealing and tabu search: a comparative study. *Engineering Applications of Artificial Intelligence*, 14(2):167–181, 2001.

[55] Shu Shi and Cheng-Hsin Hsu. A survey of interactive remote rendering systems. *ACM Computing Surveys (CSUR)*, 47(4):1–29, 2015.

[56] Andrew Lucas. Hard combinatorial problems and minor embeddings on lattice graphs. *Quantum Information Processing*, 18(7):203, 2019.

[57] Andrew Lucas. Ising formulations of many np problems. *Frontiers in physics*, 2:5, 2014.

[58] Saleh A Khawatreh. An efficient algorithm for load balancing in multiprocessor systems. *International Journal of Advanced Computer Science and Applications*, 9(3), 2018.

[59] Richard Korf. Objective functions for multi-way number partitioning. In *Proceedings of the International Symposium on Combinatorial Search*, volume 1, pages 71–72, 2010.

[60] Bruno Apolloni, Nicolò Cesa-Bianchi, and Diego De Falco. A numerical implementation of “quantum annealing”. In *Stochastic Processes, Physics and Geometry: Proceedings of the Ascona-Locarno Conference*, pages 97–111, 1990.

[61] Arnab Das and Bikas K Chakrabarti. Colloquium: Quantum annealing and analog quantum computation. *Reviews of Modern Physics*, 80(3):1061, 2008.

[62] Edward Farhi, Jeffrey Goldstone, and Sam Gutmann. A quantum approximate optimization algorithm. *arXiv preprint arXiv:1411.4028*, 2014.

[63] https://qboson.com.

[64] Zhe Wang, Alireza Marandi, Kai Wen, Robert L Byer, and Yoshihisa Yamamoto. Coherent ising machine based on degenerate optical parametric oscillators. *Physical Review A*, 88(6):063853, 2013.

[65] Takahiro Inagaki, Kensuke Inaba, Ryan Hamerly, Kyo Inoue, Yoshihisa Yamamoto, and Hiroki Takesue. Large-scale ising spin network based on degenerate optical parametric oscillators. *Nature Photonics*, 10(6):415–419, 2016.